

# Parallelization of Transport-Theory based Optical Tomography Algorithms by Domain Decomposition

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**Abstract:** We implemented and tested a domain decomposition method that provides a suitable framework for parallelization of optical tomographic imaging codes based on the frequency-domain equation of radiative transfer. This leads to substantial reduction in memory requirements and increased computation speed.

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## 1. Introduction

Many application in optical tomography rely on the accurate modeling of light propagation with the equation of radiative transfer (ERT). Prevalent examples are preclinical studies involving rodents or imaging of finger joints for the detection of arthritis. In these cases, the tissue volumes considered are relatively small and high absorbing or void-like, low-scattering regions can be found inside the tissue. Consequently the diffusion approximation to the ERT is often compromised. Tomographic algorithms that are based on the ERT, however, are known to be much slower than diffusion algorithms and require much more memory [1], because of the discretization of both the spatial and angular components of the ERT. These drawback have prevented a broader dissemination and application of these codes so far. Therefore novel approaches that accelerate the convergence of ERT-based codes and reduce the memory requirements are highly desirable. So far only two studies have been presented that looked into parallelization of optical tomographic imaging codes; Hielscher and Bartel [2] suggested to distribute forward calculation that originate from different sources positions onto different processors and Holbake et al [3] distributed calculations for different source-modulation frequencies onto different processors. None of these papers considered parallelization beyond distributing forward calculations onto different processors.

In this paper we present a domain decomposition method that provides an effective means to further parallelize optical tomographic codes, to achieve even higher computation speeds and lower memory requirement. The domain-decomposition method solves a boundary value problem by splitting the domain into many smaller boundary value problems. We implemented this approach both for the forward RTE problem and gradient-based iterative inverse problem using the Portable, Extensible Toolkit for Scientific Computation (PETSc) software package [4]. In Section 2 we define the discretization of RTE and its numerical error. In Section 3, the details of implementation of the domain decomposition method is given and the ability of the parallelization algorithm is measured.

## 2. Frequency-Domain Radiative Transport Equation

The frequency-domain ERT that describes the photon density in the phase space, i.e., as a function of position  $\mathbf{x} \in \mathcal{D} \subset \mathbb{R}^3$  and direction  $\theta \in S^2$  (unit sphere of  $\mathbb{R}^3$ ) is

$$\begin{aligned} \left( \frac{i\omega}{v} + \theta \cdot \nabla + \mu_t(\mathbf{x}) \right) u(\mathbf{x}, \theta) - \mu_s(\mathbf{x}) \int_{S^2} k(\theta \cdot \theta') u(\mathbf{x}, \theta') d\theta' &= 0 && \text{in } \mathcal{D} \times S^2 \\ u(\mathbf{x}, \theta) &= q(\mathbf{x}, \theta) && \text{on } \Gamma_-. \end{aligned} \quad (1)$$

Here  $i = \sqrt{-1}$ ,  $v$  is the speed of light in the medium, and  $\omega$  is the source modulation frequency. The parameter  $\mu_t = \mu_a + \mu_s$ , with  $\mu_a$  and  $\mu_s$  being the absorption and scattering coefficient, respectively. Frequency dependent  $u(\mathbf{x}, \theta)$  is the radiance at position  $\mathbf{x}$  ( $\mathbf{x} \in \mathcal{D}$ ) traveling in direction  $\theta$  with the unit of  $Wm^{-2}sr^{-1}$ .  $q(\mathbf{x}, \theta)$  is source with the unit of  $Wm^{-3}sr^{-1}$  defined on the boundary set:  $\Gamma_{\pm} = \{(\mathbf{x}, \theta) \in \partial\mathcal{D} \times S^2 \text{ s.t. } \pm \theta \cdot \nu(\mathbf{x})\}$ . with  $\nu(\mathbf{x})$  the outward unit normal to the domain at  $\mathbf{x} \in \partial\mathcal{D}$ . The ‘‘collision’’ kernel  $k(\theta \cdot \theta')$ , describes the probability that photons traveling in direction  $\theta'$  scatter into direction  $\theta$ .

A detailed description of the discretization of the frequency-domain RTE with a finite volume method is provided in reference [1]. Briefly, with the combination of the discrete ordinates method for the angular variable, a finite-volume discretization with step scheme method for the space variable, and an appropriate boundary condition, the

RTE can be converted to the an algebraic equation. Solving equation (1) results in a value for the radiance  $u$ . In the optical tomography experiments, however, one typically measures the outgoing flux, which is given by:  $J(\mathbf{x}_d) = \int_{\Gamma_+} \theta_d \cdot \mathbf{v}(\mathbf{x}_d) u(\mathbf{x}_d, \theta_d) d\theta_d$ . Here  $\mathbf{x}_d$  is the position of detector.

In ERT-base simulation, the discretization requirement is controlled by the physical behavior of the photon propagation in the tissue, which is governed by tissue optical properties. Furthermore, using the discrete ordinate method also requires a minimum number of discrete ordinate direction to eliminate the obvious ray effect. We can define the relative error for a discretized grid size and the relative error for the discrete ordinates as:

$$\sigma_x = \sqrt{\frac{1}{N_d} \sum_{i=1}^{N_d} \frac{|J_i^{(m,k)} - J_i^{(m+2,k)}|}{|J_i^{(m+2,k)}|}} * 100\%; \text{ and } \sigma_\theta = \sqrt{\frac{1}{N_d} \sum_{i=1}^{N_d} \frac{|J_i^{(m,k)} - J_i^{(m,k+2)}|}{|J_i^{(m,k+2)}|}} * 100\%. \quad (2)$$

Here  $m$  is the number of discretized elements in per transport mean free path (tmfp), given as the length  $1/(\mu'_s + \mu_a)$ .  $k$  is the number of number of discrete ordinates in SN $k$ . As an example, we estimate numerical errors in  $2cm \times 2cm$  square computational domain with typical tissue optical properties. We choose an anisotropic scattering media with  $g = 0.9$  and the absorption and scattering coefficients  $\mu_a = 0.5cm^{-1}$  and  $\mu_s = 50.0cm^{-1}$  respectively. The discretized grid's errors and discrete ordinate's errors are plotted in Fig. 1(a) and 1(b). If one wants to achieve an error of less than  $\sigma_x$  from the Fig. 1, the grid should be  $dx < 10 \text{ tmfp}$  and SN at least equal 24 (48 discrete ordinates) for this 2D problem. This will result in a linear algebraic system with approximately  $4.2 * 10^6$  unknowns. If we consider a 3D case with similar optical properties and  $2cm \times 2cm \times 2cm$  domain, a linear system with approximately  $2.0 * 10^9$  unknowns has to be solved. Such large systems of linear equations can currently not be handled by most single processor computers, because of memory limitations and typical computation speeds.

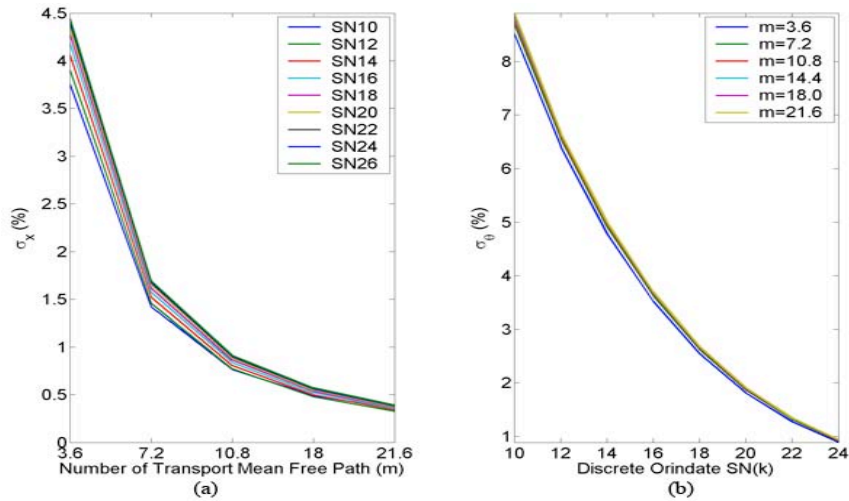


Figure 1: The simulation errors decrease with (a).the number of discretized elements per tmfp; (b).the number of discrete ordinates.

### 3. Implementation of Domain Decomposition Method and Scalability

#### 3.1. Domain Decomposition in Forward Problem and the Scalability of the Forward Algorithm

The purpose of the parallization is to partition one big numerical problem into many smaller ones, which can be handled by many single processors that act in parallel. The first step to realize the memory distribution is to divide and scatter the mesh on which the computation is performed. After settling down the submesh on each processor, the most difficult and important part of this parallization algorithm is to deal with the submesh's boundary nodes, whose information has to be available by the neighboring or related processors. The exchanging efficiency of boundary nodes' information determines the effectiveness of the parallelization algorithm. With all the nodes information available, a global (handled by mutli-processor) problem becomes a local (like a single- processor) one, which all the available linear algebraic solver can be adopted. Finishing solving the matrix equations, the solution collection and output can be handled by the popular MPI functions.

The effectiveness of the parallelization algorithm can be characterized by its scalability, which is defined as the accelerated time versus the number of the processors. We have extensively tested the scalability of the forward algo-

rithm on our lab cluster, which is a four-nodes dual-processors cluster with 226 GHz IntelXeon CPU; 2 GB RAM controlled by each node; and 1Gbit ethernet for the communication between the nodes. As an example we discuss here a case that uses the same geometry set and optical properties as the case used for the error estimation Section 2. We used  $SN=26$  and  $dx = 18$  tmfp. Fig. 2(a) plots the speed increase as a function of the number of the processors used. One can see that with 8 processors the computation speed can be increased by a factor of 6, which constitutes excellent scalability.

### 3.2. Domain Decomposition in Inverse Problem and the Scalability of the Inverse Algorithm

Details concerning the implementation of the gradient-based iterative inverse algorithm on a single computer can be found in reference [5]. With the Quasi-Newton limited memory BFGS algorithm, the inverse algorithm includes solving the forward problem, solving the adjoint problem and performing a superlinear convergent optimization. The domain decomposition method applied to the forward problem introduced in section 3.1 can be exactly applied to the adjoint problem. The main issue in the implementation is to parallelize the optimization algorithm. Focusing on reconstructing the absorption coefficient only, one can perform the same steps to distribute the discretized absorption parameters on multi-processors. However, because of the many evaluations need to be estimated globally, the scalability of inverse algorithm is not quite as good as the scalability of the forward problem. One can see in Fig. 2(b) that using 8 processors will increase the computation speed by approximately a factor of 5.

Overall, we have shown that using our domain-decomposition approach allows to increase the computational speed proportional to the number of processors available. Given that modern supercomputers, such as the Brookhaven/Stony Brook IBM Blue Gene supercomputer, make up to 36,000 processors available, the study of highly complex optical systems with the equation of radiative transfer becomes feasible.

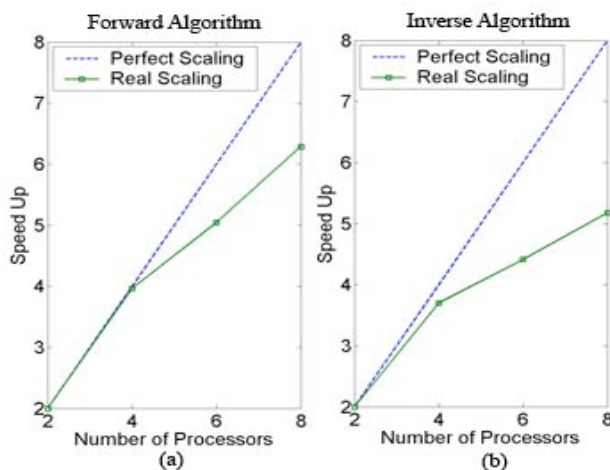


Figure 2. The strong scalability of forward algorithm (a) and inverse algorithm(b) on the lab cluster.

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